Neo Science Peer Reviewed Journal
Volume 4, Dec. 2022 ISSN (E): 2949-7701
www.neojournals.com
==

ABOUT ONE TASK OF YU. V. PROKHOROV

M. U. Gafurov
Doctor of Physical and Mathematical Sciences, Professor
Tashkent State Transport University

Abstract:
An asymptotic estimate of the convergence velocity in the law of repeated logarithm in the form of convergence of series from the probabilities of large evasions is obtained.

Keywords:
the law of repeated logarithm, the convergence of the series, the number of outputs, the normal law, the speed of convergence in the central limit theorem

1. Introduction. Let {}be independent equally distributed random variables (s.v.) with zero mean and finite variance$X, X_n; n \geq 1$

$$EX = 0, \quad EX^2 = 1$$

Put

$$S_0 = 0, \qquad S_n = \sum_{k=1}^{n} X_k, \qquad n = 1,2 \dots$$

The classical Hartmann–Wintner repeated logarithm law is known [ ]:

$$\lim_{n\to\infty} \pm \frac{S_n}{\sqrt{2n \ln \ln n}} = 1$$

The task of studying the rate of convergence in the above ratio is of great interest, and at this time a large number of articles have been published on this subject, the results of which have various applications in interpretation.

Analysis of the results shows that research on this task was conducted in two directions:

A. For a given function, estimating probabilities of a form:$H(x)$

$$P_n = P\{S_k \geq H(k) \text{ для некоторого } k \geq n\} \text{ при } n \to \infty$$

In the works of D. Darling and G. Robbins [ ], G. Robbins and D. Sigmund [ ] in the assumption established, $EexphX < \infty$ для $|h| < h_0$

A what

(a) There is a sequence

$H(n) \sim \sqrt{2n \ln \ln n}$ such that with $n \to \infty$

$$P_n = O\left(\frac{1}{\ln \ln n}\right);$$

c) if , for some , then$H(n) \sim \sqrt{2an \ln \ln n}\ a > 1$

at $n \to \infty$

==

===
$$P_n = O\left(\frac{1}{\ln^{a-1} n}\right)$$
The most accurate results in terms of descending order were established by W. Strassen [ ]. He proved that if the function is such that$P_n EexphX < \infty$ для $|h| < h_0\ (h_0 > 0) H(x)$
$x^{-1/2}H(x) \uparrow, H(x) \le x^c, c < 3/5$
$$\frac{H'(x)}{H(y)} \to 1 \text{ при } \frac{x}{y} \to 1\text{, то при } n \to \infty$$
$$P_n \approx \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{H'(t)}{\sqrt{t}} \exp\left\{-\frac{H^2(t)}{2t}\right\} dt$$
Strassen's research continued in the works of T.L. Lai V.S. Chow [ ], T.L. Lai and K. Lan [ ], M.U. Gafurov and A.D. Slastnikov [ ], M.U. Gafurov []. In particular, it is noted that it follows from the overall results [ ] that if , then$H(x) \approx \sqrt{(2+\varepsilon)\ln\ln n}$
$$P_n \le C\sqrt{\ln\ln n}\ln^{-\frac{\varepsilon}{2}} n, \qquad C > 0, \forall \varepsilon > 0$$
which is the best, in the sense of the order of descending to zero probability P_n.
B. Associate the non-negative, monotonously increasing function and non-negative function with the series$H(x) f(x)$
$$R = \sum_{n=1}^{\infty} \frac{f(n)}{n} P(|S_n| > H(n)).$$
It should be noted that with proper choice of function and convergence of the series characterizes the rate of convergence in the law of repeated logarithm. For special cases, function and convergence were investigated by J. S. Miller. Davis [ ].$f(x)H(x)Rf(x)H(x)R$
He proved that if the conditions are met, the following statements are equivalent:$EX = 0, EX^2 \ln|X| \ln\ln|X| < \infty$
$$\sum_{n=1}^{\infty} \frac{\varphi^2(n)}{n} P\{|S_n| > \sqrt{n}\varphi(x)\} < \infty$$
and
$$\int_1^\infty \frac{\varphi^2(x)}{x} e^{-\frac{\varphi^2(x)}{2}} dx < \infty$$
where $\varphi(x) > 0,\ \ \varphi(x) \uparrow, \quad x \in [1, \infty)$.
This result was summarized and reinforced in the work of M.U. Gafurov [ ] for a broad class of functions and and distribution functions.$f(x)H(x)X$
Let be strictly increasing functions defined on satisfying conditions:$f(x)\varphi(x)[1,\infty)$
$$\frac{f(x)}{\varphi^2(x)} \uparrow, \quad \frac{f(x)}{\varphi^3(x)} \downarrow, \qquad x > x_0, \qquad x_0 > 0 \qquad\qquad (\cdot)$$
===

===

Let's put and denote the function inverse of .$H(x) = \sqrt{x}\varphi(x) H^{-1}(x) H(x)$

Theorem 1 [ ]. Let the conditions be met,

$$EX = 0, EX^2 = 1 \text{ и } E\left[H^{-1}(|X|) f[H^{-1}(|X|) \ln|X|]\right] < \infty \qquad (\cdot)$$

Then

$$R < \infty \Leftrightarrow \int_1^\infty \frac{f(x)}{\sqrt{x} H(x)} \exp\left\{-\frac{H^2(x)}{2x}\right\} dx < \infty$$

In the same work, a special case is highlighted when

$$\varphi(x) = \sqrt{(2+\varepsilon)\ln\ln x}, \quad f(x)$$

Then, it is not difficult to understand that , the condition takes the form

$$H^{-1}(x) = \frac{x^2}{(2+\varepsilon)\ln\ln x} E\left[H^{-1}(|X|) f[H^{-1}(|X|) \ln|X|]\right] < \infty \; EX^2 \ln|X| < \infty.$$

Let's introduce the functionality into consideration

$$\lambda_\varepsilon = \sum_{n=3}^{\infty} \frac{\ln\ln n}{n} I\left(|S_n| > \sqrt{(2+\varepsilon) n \ln\ln n}\right),$$

where is the event indicator , which is the "weighted number" of outputs of the random wandering trajectory beyond the two-way boundary $I(A) A \pm (2+\varepsilon) n \ln\ln n.$ It is easy to understand that under p.n. the naturally occurring problem of asymptotic behavior in dependence was solved for the first time in [ ]:$\varepsilon \downarrow 0 \quad \lambda_\varepsilon \uparrow \infty \lambda_\varepsilon \varepsilon \downarrow 0$

Theorem 2 [ ]. Let$EX = 0, EX^2 = 1, EX^2 \ln|X| < \infty.$

Then

$$\lim_{\varepsilon \downarrow 0} \varepsilon^{3/2} E\lambda_\varepsilon = \sqrt{2}$$

The statements of these theorems 1,2 and the methods of their proof attracted the attention of specialists in Germany, China, Sweden and others, and they stimulated the emergence of a number of new refinements and generalizations in this direction [ ] ... [ ].

Regarding the approval of theorem 2, we also add that back in 1981 at a seminar on probability theory of the Steklov Mathematical Institute of the Russian Academy of Sciences, Academician Yu.V. Prokhorov expressed the task of finding further members of the asymptotic representation. However, this task has not yet been solved. It is to the solution of the task that the present work is devoted.$E\lambda_\varepsilon$

Theorem 3. Under the conditions of theorem 2 at $\varepsilon \downarrow 0$

$$\varepsilon\sqrt{\varepsilon} E\lambda_\varepsilon = \sqrt{2} - \frac{3\sqrt{2}}{4}\varepsilon + o(\varepsilon)$$

===

References

1. Darling D. A., Robbing H. “Iterated Logarithm Inequalities” Proceedings of the National Academy of Sciences, USA, 1967, 1188-1192;
2. Robbins H., Siegmund D. “Iterated Logarithm and Related Procedures” Mathematics of the Decision Sciences, 2-lectures in Appl. Math. Amer. Math. Soc. 1968, 57, 267-279;
3. Strassen V. K. “Almost Sure Behavior of Sum of Independent Variables and Martingales” Proc. 5th Berkley Symp. Math. Stat. Prob., 1967, v. 2, part 1, 315-343;
4. Chow Y. S., Lai T. L. “Some one-sided theorems on the tail distribution of sample sums with application to the last time and largest excess of boundary crossing” ransactions of the American Mathematical Society, 1975, 208, 51-72;
5. Lai T. L., Lan K. K. “On the last time the numbers of the boundary crossing related to the law of large numbers and the law of the iterated logarithm” Z. Warh. Verw. Geb., 1976, 34, 1, 59-71;
6. Gafurov M. U., Slastnikov A. D. "Some problems of exit of random wandering beyond the curvilinear boundary and large deviations" Theor. Ver. and application. 1987, vol. 32, vol. 2, 327-348;
7. Gafurov M. U. "Generalized variants of "r-fast" convergence of V. Strassen" Intelligent systems, 12, 2016, volume 20, vol. 1, Lomonosov Moscow State University Publishing House, 25-29;
8. Devis “Convergence rates for the law of the iterated logarithm” Ann. Math. Stat., 39, 1968, 1479-1485;
9. Akmal Mukhitdinov, Kamoliddin Ziyaev, Method of cycles by synthesis.2021.// Conference Network E3S. 6/4Volume No.264.-P.01033
10. Anvarovich M.A., Zukhritdinovich Z.K. Method for assessing the energy efficiency of regulated driving cycles. // Review of European Science, 2016
11. Kamoliddin Zukhritdinovich Ziyaev. Method for determining transport intensity in urban conditions20224/9.Volume No.264.-P.111-114
12. K Ziyaev The Appeal is at the Center of Youth Issues .// Academic Journal of Digital Economics and Stability, 2021 Том №6.-С.-36-38
13. K.Z . Ziyayev -Method of quantitative research of navoi city on the basis of choice of traffic flow.// The Scientific Journal of Vehicles and Roads, 2021 Том №2.-С 27-36
14. Gafurov M. U. “On the estimate of the rate of convergence in the law of iterated logarithm in probability theoty and mathematical statistics” (Tbilisi, 1982) in Lecture Notes in Math., vol. 1021, Springer-Verlag, Berlin, 1983, pp. 137-144;
15. Deli Li, Bao-Em Nguyen, Andrew Rosalsky “A supplement to precise asymptotics in the law of iterated logarithm” J. Math. Anal. Appl. 302 (2005), 84-96.