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ON THE CONDITIONS OF CONTROLLABILITY FOR DIFFERENTIAL INCLUSION

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Abstract:	Keywords:
In this paper for one class of differential inclusion the problem of controllability is researched. For this system the necessary and sufficient conditions of controllability relatively given terminal set are obtained.	dynamic control system, differential inclusion, terminal set, conditions of controllability.

1. Introduction

Differential inclusion has wide applications in the theory of dynamic system management, differential games and other aspects of modern mathematics. In the theory of optimal control, the essential characteristic of dynamic control systems is the property of control, which indicates the possibility of transferring the trajectory system from the initial state to the desired final state. Poetomu problema upravlyaemosti actualna dlya dynamic system, opisyvaemyx lichnymi class differential inclusion.

And the theory of management of great interest presents problems of conditional and relative management, as well as conditions of local management. Special features such as local control and control and good for differential inclusion analysis and work. The property of the controllability of differential inclusion with consideration of the structure of a given set of terminal state and controllable ensemble trajectory of differential inclusion and parameters is studied.

2. Statement of the problem.

For dynamical systems, an important problem is the possibility of translating a system described by a differential inclusion

$$\dot{x} \in F(t, x),\ t \geq t_0, \quad x(t_0) = x_0 \in Z, \tag{1}$$

from a set of initial states $Z \subset R^n$ to a given terminal set $M \subset R^n$ on some finite time interval $T = [t_0, t_1]$.

By admissible trajectories of the considered system (1) we mean every absolutely continuous n-vector function $x = x(t)$ that satisfies a given differential inclusion almost everywhere on a certain time interval $T = [t_0, t_1]$.

Definition. We will say that the differential inclusion (1) is controllable from the initial set $Z \subset R^n$ to the terminal set M on the time interval $T = [t_0, t_1]$, if there is an absolutely continuous trajectory $x(t)$, such that $x(t_0) \in Z$, $x(t_1) \in M$.

The controllability of the differential inclusion from the initial set $Z \subset R^n$ to the terminal set M is equivalent to the fact $X(t_0, t_1, Z, F) \cap M \neq \varnothing$ that at some moment $t_1 > t_0$, where $X(t_0, t_1, Z, F)$ is the reachability set of the dynamical system (1) at time $t_1 > t_0$, i.e., the set of ends $x_1 = x(t_1)$ of all possible absolutely continuous trajectories $x = x(t)$ with the initial condition $x(t_0) \in Z$.

We use the following notation:

$\Omega(R^n)$ - the set of all compact (i.e. closed and bounded) subsets from the Euclidean space R^n; $co\Omega(R^n)$ is the set of all convex compact subsets from R^n; $\|X\| = \sup_{\xi \in X} \|\xi\|$ is the norm of a bounded set $X \subset R^n$; $X + Y = \{z : z = x + y, x \in X, y \in Y\}$ is the algebraic sum of the sets X and Y from R^n; $\lambda X = \{z : z = \lambda x,\ x \in X\}$ is the product of a set by X a number $\lambda \in R^1 = (-\infty, +\infty)$; R^{nxn} is the space of real $n \times n$-matrices $A = (a_{ij}, i, j = \overline{1, n})$ with the norm $\|A\| = (\sum_{i,j=1}^{n} a_{ij}^2)^{\frac{1}{2}}$; $L_1(T)$ - space of Lebesgue T integrable functions $g = g(t)$; $c(X, \psi) = \sup\{(x, \psi) :\ x \in X\}$ is the support function of the compact set $X \subset R^n$.

3. Properties of the reachable set of a linear differential inclusion.

Consider the linear differential inclusion

$$\dot{x} \in A(t)x + B(t),\ t \geq t_0,\ x(t_0) \in Z, \tag{2}$$

where $A : R^1 \to R^{nxn}, B : R^1 \to \Omega(R^n)$. We will assume that the elements of the matrix $A(t)$ are measurable on $T = [t_0, t_1]$ and $\| A(t) \| \leq a(t)$, where $a(\cdot) \in L_1(T)$ and $t \to B(t) \in \Omega(R^n)$ the multivalued mapping is measurable on the interval $T = [t_0, t_1]$ and $\| B(t) \| \leq b(t)$, where $b(\cdot) \in L_1(T)$. According to the accepted notation, $X(t_0, t_1, x_0, A, B)$ is the reachability set of the differential inclusion (2).

Denote by the $\Phi_A(t, \tau)$ fundamental matrix of solutions to the equation $\dot{x} = A(t)x,\ t \in T$. It is well known from the results of the theory of differential equations that, for each integrable function $b : T \to R^n$, an absolutely continuous solution of the initial problem

$$\dot{x} = A(t)x + b(t), t \in T, x(t_0) = \xi$$

represented through the Cauchy formulas

$$x(t) = \Phi_A(t, t_0)\xi + \int_{t_0}^{t} \Phi_A(t, \tau) b(\tau) d\tau, t \in T. \tag{3}$$

Using formula (3) and Filippov's lemma on implicit functions [1,3,15], one can obtain a representation of the reachability set of differential inclusion (2) in terms of the fundamental matrix $\Phi_A(t, \tau)$ and multivalued mapping $B : T \to \Omega(R^n)$

Lemma. The correct formula is:

$$X(t_0,t,Z,A,B)=\Phi_A(t,t_0)Z+\int_{t_0}^{t}\Phi_A(t,\tau)B(\tau)d\tau,\ t\in T. \qquad (4)$$

Corollary 1. Let is $Z\subset R^n$ a convex set. Then the set $X(t_0,t,Z,A,B)$ is convex for all $t\in T$. If $Z\in co\Omega(\grave{R}^n)$, then the set is a convex $X(t_0,t,Z,A,B)$ compact set R^n from for all $t\in T$.

Corollary 2. The support function of the set $X(t_0,t,Z,A,B)$ is calculated by the formula:

$$c(X(t_0,t,Z,A,B),\psi)=c(\Phi_A(t,t_0)Z,\psi)+\int_{t_0}^{t}c(\Phi_A(t,\tau)B(\tau),\psi)d\tau. \qquad (5)$$

4. Controllability conditions.

Consider the following function:

$$\mu(\psi)=c(\Phi_A(t_1,t_0)Z,\psi)+\int_{t_0}^{t_1}c(\Phi_A(t_1,\tau)B(\tau),\psi)d\tau+c(M,-\psi),\psi\in R^n. \qquad (6)$$

Theorem. The differential inclusion (2) is controllable from a convex initial Z set to a convex terminal set M on a time interval $T=[t_0,t_1]$ if and only if

$$\inf_{\|\psi\|=1}\mu(\psi)\geq 0. \qquad (7)$$

Proof. As noted above, the controllability of the differential inclusion (2) from the initial set Z to the terminal set M on a time interval $T=[t_0,t_1]$ is equivalent to the relation

$$X(t_0,t_1,Z,A,B)\cap M\neq\varnothing$$

The latter takes place if and only if

$$0\in X(t_0,t_1,Z,A,B)-M. \qquad (8)$$

By Corollary 1, the set $X_T(t_1,Z,A,B)$ is convex, and by the given condition M it is a convex set, and therefore their difference is also convex. Therefore, relation (8) is equivalent to the inequality

$$c(X_T(t_1,Z,A,B),\psi)+c(M,-\psi)\geq 0,\forall\psi\in R^n,\|\psi\|=1$$

Using formulas (4) and (5) from the last relation we obtain:

$$c(\Phi_A(t_1,t_0)Z,\psi)+\int_{t_0}^{t_1}c(\Phi_A(t_1,\tau)B(\tau),\psi)d\tau+c(M,-\psi)\geq 0,\ \|\psi\|=1$$

Now, taking into account (6), the last relation is written in the form (7). The theorem has been proven.

Corollary 3. If the differential inclusion (2) is controllable from a convex compact initial set Z to a convex compact terminal set M on time interval $T=[t_0,t_1]$ and $\Phi_A(t_1,t_0)Z\cap M=\varnothing$, then for all directions $\psi^*\in R^n,\|\psi^*\|=1$ strictly separating sets $\Phi_A(t_1,t_0)Z$ and M hyperplanes, the inequality

$$\int_{t_0}^{t_1} c(\Phi_A(t_1,\tau)B(\tau),\psi^*)d\tau > 0. \quad (9)$$

Indeed, the condition implies the existence of $\Phi_A(t_1,t_0)Z \cap M = \varnothing$ at least one vector $\psi^* \in R^n, \|\psi^*\| = 1$ such that the relation holds

$$c(\Phi_A(t_1,t_0)Z,\psi^*) + c(M,-\psi^*) < 0$$

The latter means that $\Phi_A(t_1,t_0)Z$ and M are strictly separated by a hyperplane having direction $\psi^* \in R^n, \|\psi^*\| = 1$. Now, if we assume that condition (9) is not satisfied, then we obtain the following inequality:

$$\inf_{\|\psi\|=1} \mu(\psi) \le c(\Phi_A(t_1,t_0)Z,\psi^*) + c(M,-\psi^*) + \int_{t_0}^{t_1} c(\Phi_A(t_1,\tau)B(\tau),\psi^*)d\tau < 0.$$

But this contradicts the necessary controllability condition (7).

Corollary 4. If

$$\inf_{\|\psi\|=1}[c(\Phi_A(t_1,t_0)Z,\psi) + c(M,-\psi)] + \inf_{\|\psi\|=1}\int_{t_0}^{t_1} c(\Phi_A(t_1,\tau)B(\tau),\psi)d\tau \ge 0, \quad (10)$$

then the differential inclusion (2) is relatively controllable from a convex initial set Z to a convex terminal set M on the time interval $T = [t_0, t_1]$.

In fact, because

$$\mu(\psi) \ge \inf_{\|\psi\|=1}[c(\Phi_A(t_1,t_0)Z,\psi) + c(M,-\psi)] + \inf_{\|\psi\|=1}\int_{t_0}^{t_1} c(\Phi_A(t_1,\tau)B(\tau),\psi)d\tau$$

then by virtue of (10) we obtain the sufficient condition (7).

5. Conclusion

The paper considers the question of controllability of a dynamical system for one class of differential inclusions. Using the properties of the reachability set of a differential inclusion, we prove necessary and sufficient conditions for the controllability of the considered system from a given initial set to a terminal set. The resulting condition (7) is a controllability criterion for a dynamic system described by differential inclusion (2). The corresponding corollaries are deduced from the obtained results.

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